Time: 2 hours 30 minutes

Total marks: 30

Answer all questions. No marks will be awarded in absence of proper justification.

- (1) Consider the following vectors in  $\mathbb{R}^4$ :  $v_1 = (1, 2, 1, -1)^t, v_2 = (2, 4, 1, 1)^t, v_3 = (-1, -2, -2, -4)^t, v_4 = (3, 6, 2, 0)^t.$ Find a basis for the span of  $\{v_1, v_2, v_3, v_4\}.$  (6)
- (2) Extend the vector  $v = (1, 1, ..., 1) \in \mathbb{R}^n$  to a basis of  $\mathbb{R}^n$ . (6)
- (3) Let V = ℝ<sup>2</sup> and W = ℝ<sup>3</sup>, T : ℝ<sup>2</sup> → ℝ<sup>3</sup> be the linear transformation given by T(x<sub>1</sub>, x<sub>2</sub>)<sup>t</sup> = (2x<sub>1</sub> - 3x<sub>2</sub>, x<sub>1</sub>, 5x<sub>1</sub> + x<sub>2</sub>)<sup>t</sup>.
  (i) Describe the nullspace and the range of T.
  (ii)Find the matrix of T with respect to the ordered bases B<sub>1</sub> = ((1, -1)<sup>t</sup>, (4, 0)<sup>t</sup>) and B<sub>2</sub> = ((1, 1, 0)<sup>t</sup>, (1, 0, 1)<sup>t</sup>, (0, 1, 1)<sup>t</sup>) of ℝ<sup>2</sup> and ℝ<sup>3</sup> respectively. (6+6)
- (4) State and prove rank-nullity theorem.

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(6)
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